have a compendium that is likely to remain a standard work of reference for many years to come.

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The Oxford Handbook of the History of Mathematics
Edited by Eleanor Robson and Jacqueline Stedall. Oxford (Oxford University Press). 2009. ISBN: 978-0-19-921312-2. vii + 918 pp. £85.00

The sheer scope of the thing exhilarates. A first glance into this hefty volume promises enlightenment on mathematics in a Babylonian classroom, in medieval theology, in the Third Reich, in John Aubrey's *Brief Lives*, in Sanskrit verse, in 19th-century Naples, in astronomical observatories, in "modern" culture, in traditional Vietnam—and much, much more. Jaded indeed must be a reader who cannot find fascination somewhere in a compendium so rich and so diverse.

Eleanor Robson and Jacqueline Stedall, who will not need introducing to regular readers of this journal, have in fact assembled 36 articles spanning, as the sample above suggests, much of the globe and much of recorded history. Their roster of contributors ranges widely too: it includes, they say (p. 3), "old hands alongside others just beginning their careers, and a few who work outside academia"—for example a research associate at a textile museum. This breadth of subject-matter and of authorial expertise clearly goes to the heart of the editors' purpose. They say (p. 1) that they wish to "raise new questions about what mathematics has been and what it has meant to practise it". They urge (p. 1) that

> [M]athematics is not confined to classrooms and universities. It is used all over the world, in all languages and cultures, by all sorts of people. Further, it is not solely a literate activity but leaves physical traces in the material world: not just writings but also objects, images and even buildings and landscapes.

I suspect that few readers will find any of these propositions revelatory; but however that may be, certainly the editors' deep commitment to them gives their book much of its flavor.

In trying to impose order on their embarrassment of riches Robson and Stedall have adopted a scheme which regrettably seems neither natural nor useful. They divide the 36 papers into three precisely equal collections, tagged respectively "Geographies and Cultures", "People and Practices", and "Interactions and Interpretations", and then they partition each of these subsets into three groups of exactly four papers each. The symmetry is elegant, but a skeptic might protest that those three labels are too broad, too vague and too overlapping to make helpful signposts. Quickly, now: under which of them would one seek, let us say, Markus Asper's article on the "two cultures" of mathematics in ancient Greece? Under "People and Practices"? Why not? What topic in the history of mathematics would *not* fit comfortably under so welcoming an umbrella? Actually Robson and Stedall assign the Asper paper to "Geographies and Cultures"—and, again, why not? That too makes perfect sense. Unfortunately the editors' choices work against their expressed desire (p. 2) that readers make "connections" among the articles: so far as I can see, *none* of their four-item clusters has *any* nontrivial thematic coherence. To their credit they concede (p. 2) that many other permutations would be "possible and interesting", and anyway a reader can of course easily browse or locate by just scanning down the list—which is all that really counts.

Still, labels do convey meanings, and this fact underlies my only serious reservation about this attractive book. Robson and Stedall kick off their preface with the disarming declaration that they "hope that this book will not be what you expect", and so I am delighted to report that my own first reaction was very much in that spirit: I was astonished. For I was struck at once by an unsettling gap between the volume's title and its contents. The *Oxford English Dictionary* advises that a "handbook" is (i) "a small [!] book or treatise ...; a manual" or (ii) "a compendious book or treatise for guidance in any [of course 'some' would be better] art, occupation or study"; and all of the several other dictionaries which I consulted say more or less the same thing. Now, *some* of the articles in the present collection could reasonably qualify under this definition: they are broad surveys of particular topics in the history of mathematics, with pointers to potential future enquiry. But the great majority of these contributions are prototypical research papers, worthy in their own right but narrowly focused and closely argued—well suited, for example, to the "Regular Articles" section of this journal. Of course a scholar interested in one of these papers' subjects may get from it "guidance" for her own work, but if that were the criterion then *any* collection of such papers could count as a handbook. Taken as a whole, the Robson–Stedall compendium assuredly does *not* do for history of mathematics as a whole what the *OED* says a handbook should do for a field of study. Caveat lector: a reader who, swayed by the book's title, shells out for her own copy or seeks it out in a library is in for a big surprise.

I shall try to convey some sense of each of these essays, taking them at least roughly in the chronological order of their subjects and in natural groupings. (Each quotation is from the article under discussion.) Reaching furthest into the past is Stephen Chrisomalis's study of "number words, computational techniques, and number symbols" (p. 496). He writes that understanding the "linkages" among these, and the "functions each serves (and does not serve) will help illustrate the range of variability among the cognitive and social systems underlying all mathematics" (p. 496). This wide-ranging survey of knowledge and scholarship would be a very good candidate for a handbook legitimately so called.

Eleanor Robson reports that a building excavated at Nippur in 1952 can be supposed, by study of the more than 1400 tablets found inside, to have been a school. The great majority of the mathematical tablets taught elementary metrology and arithmetic; a large group of literary texts, part of an advanced scribal curriculum, conveyed to students, through their imagery, messages "about mathematics and the scribes' relationships to it" (p. 216). Annette Imhausen targets the pervasive "myths" (p. 781) in our perceptions of Egyptian mathematics, myths which she ascribes to the haphazard survival of sources and to an "obsolete" (p. 781) tradition of historiography (Van der Waerden, Neugebauer). Her five examples of such misapprehensions include the supposed use of Pythagorean triplets and the supposed restriction to unit fractions. Seeking insight into the Egyptians' use of mathematics in technology, Corinna Rossi proposes to reconsider the familiar sources (papyri, leather rolls, tablets) "in search of further clues" (p. 408), taking metal extraction and food production as spheres of potential application. The sources, she concludes (p. 423), may indeed "indirectly provide solutions for [technological] problems" that they do not explicitly mention. A droll bit of irony attaches to Kim Plofker's essay on ancient India: she uses the book's best prose—unfailingly clear and vigorous, and graced by delightful turns of phrase—to write about *verse*, specifically the Sanskrit metrical verse which was the "chief vehicle of mathematical learning" (p. 535). Commentaries and diagrams supplemented the verses but never fully replaced them—as if, says Plofker with typical flair (p. 536), the "purely literate mathematics of the sort that developed in the West, with its dependence on laborious descriptions of figures and equations, would have seemed to Indian mathematicians simply too, well, prosy".

Four articles deal wholly or partly with ancient Greece. Geoffrey Lloyd's fine overview of the respective Greek and Chinese answers to the question "What is mathematics?" finds (p. 25) in the two approaches a "fruitful heterogeneity": the former trying to ground the subject in self-evident axioms, the latter seeking to "expand it by extrapolation and analogy" (p. 25). This is another paper that would not be out of place in a genuine handbook. Ken Saito wrestles with the fundamental problem of approaching the Greek classics with due sensitivity. The histories of the Archimedes palimpsest, of textual criticism of the *Elements*, and of readings of particular Euclidean propositions teach that "no interpretation can remain complete or definitive" (p. 825). Markus Asper sets out the contrasts—in procedures, in texts, in community status—between the "two cultures" (p. 107) in Greek mathematics, and suggests (p. 129) that the unusual features of the theoretical tradition evolved as "markers of differentiation", intended to distance it from the "social and epistemic" aspects of the practical side. David Gilman Romano explores the use of mathematics by Greek and Roman surveyors in and around Corinth, taking as his examples the curved starting line of a racecourse (5th century B.C.), whose design aimed to ensure fairness, and the agrimensores' division (2nd century B.C. to 1st century A.D.) of rural areas into regular units, mainly to facilitate the collection of taxes.

Christopher Cullen's essay on ancient China goes back to basics: "Can we identify an activity ... with a family resemblance to what would nowadays be called 'mathematics'?" (p. 593). To this end he examines the work of particular experts in *suan*, the theory and practice of the use of counting rods. He concludes (p. 609) that until the end of the Han dynasty *suan* was "an essential skill" but not a "major focus of intellectual attention". The other two papers on China assess migrations of mathematics, respectively from and to the Middle Kingdom. Andrei Volkov describes the transmission of mathematical expertise from China to Vietnam, which may go back to the first millennium A.D. He concentrates on three aspects: "the use of counting instruments, the use of the Vietnamese written language Nôm in mathematical treatises, and the Vietnamese state mathematical examinations" (p. 159). Catherine Jami widens the standard account of the Jesuit introduction of mathematics to China by looking at material other than Euclid and by viewing the encounter as a "complex interaction" (p. 79) rather than as one-way. Especially important among late-17th-century consequences were a Chinese-Western synthesis produced by Mei Wending and the "appropriation" (p. 72) of Western science by the Kangxi emperor.

Our Islamic heritage occupies three papers. Sonja Brentjes describes patronage of the mathematical sciences in Islamic society—mostly at courts and by "wealthy urban groups" (p. 305) before the 12th century, then increasingly in "endowed teaching institutions" (p. 312). Her discussion includes the "rhetoric" of patronage (p. 314), the forms of remuneration, and the mathematical and scientific "outcomes" (p. 320). Brian Spooner and William L. Hanaway describe *siyaq*, a system of numerical notation developed in the 7th or 8th century and used in "Persianate" regions (p. 429) into the 20th. Until the "jolt of colonialism" (p. 444) toward 1900 this was largely the preserve of professional communities of scribes working in government, land management and trade. Carol Bier raises, and inclines to answer in the affirmative, the question whether Islamic art's "emphasis on geometry and surface" (p. 827) intentionally expresses "emergent mathematical ideas in the context of their creation" (p. 828)—for example the concept of an algorithm; she points also to the possible influence of contemporary philosophy. Conversely, she suggests, the geographical diffusion of textile patterns may have spread the "mathematical knowledge" (p. 845) which they embodied.

Sabine Rommevaux's title promises (p. 687) "three case studies" on the medieval translations of the *Elements*; these turn out to deal respectively with pyramids and prisms, with the irrationality of magnitudes, and with ratios and proportions. Distortions of the Greek original introduced in each of these areas important modifications of Euclid's definitions and theories, and these changes were duly handed down to the Renaissance in the standard text produced by Campanus de Novare in the 1260s. Mark Thakkar discerns much of mathematical interest in 14th-century writers responding to, but going far beyond, Peter Lombard's *Sententiae* (1150s), a compilation of opinions of the Fathers and later theologians. Commentators addressed (inter alia) the ontological status of numbers, the divisibility of magnitudes, and the riddles posed by infinite sets. Awareness of their work, Thakkar says, should correct the widespread tendency of historians of mathematics to dismiss or disparage the scholastics.

Latin America is a shared backdrop for two otherwise very different articles. Drawing on Foucault and Max Weber, Gary Urton investigates the degree to which social authority and power can stem from possession of techniques (like accounting) that manipulate numbers. His test cases are the double-entry book-keeping developed in Renaissance Europe, the administrative use of "khipus" (knotted cords) in the pre-Columbian Andes, and the confrontation of these two traditions after the Spanish conquest. Mathematics, says Urton by way of summary (p. 51), "may be made to serve, although it itself is not responsible for giving rise to, regimes of power". Carrie Brezine detects in the weaving of textiles "a multitude of mathematical problems ranging from arithmetical to abstract symbolical manipulation" (p. 469). She follows an instructive primer on fabrics with a comparison of the techniques of weaving on, respectively, the "floor" loom of European practice and the "backstrap" loom used in South America. In the latter context, she says (p. 490), the resulting textiles hint at conceptions of "space, number and symmetry"; in the former, they reveal the "geometric understanding needed to achieve complex patterning within the restrictions" (p. 490) imposed by the machine.

Volker R. Remmert describes 17th-century attempts to legitimize and praise the multiplying subdisciplines of the mathematical sciences through the choice of images to serve as the frontispieces of books. Pictorial evocations of the antiquity of these pursuits—with Archimedes as the favorite subject—and of their nobility and their usefulness were considered especially persuasive. John Denniss chronicles the changes in English textbooks of arithmetic between 1500 and 1900. Prominent were a broadening of content, the introduction of "ready reckoners", and a shift of the targeted readership from adults toward children. Youngsters lacking access to printed books often made their own "manuscript textbooks" (p. 458), of whose pages this article contains several charming reproductions.

Three contributors focus on the lively world of 17th-century England. Jacqueline Stedall identifies "networks of informal mathematical communication" involving "not only practitioners but also patrons and interested bystanders" (p. 134), and often taking up issues outside the contemporary mainstream—for example a method worked out by Thomas

Harriott for interpolating tables using constant differences. These relatively obscure undercurrents, Stedall argues (p. 150), "offer us a new perspective" on the age. Kate Bennett sketches the sources, collaborations, aims, and strategies behind the mathematicians' biographies which (she says) are at the "core" (p. 329) of John Aubrey's captivating *Brief Lives*. A common theme in those life-stories is the advantage conferred by an early education in mathematics; a long section here stresses the claimed possession of mathematical skill as vital to William Petty's carefully constructed self-representation as "faber fortunae", maker of his own fortune. Benjamin Wardhaugh discusses the "range of issues that arose when scholars attempted to make sense in the new seventeenth-century context of the mathematical musical tradition they had inherited" (p. 640), which of course went back to the Pythagoreans. Experiments performed by the Royal Society on four "instruments" designed for the mathematical study of music illustrate that no "consensus" was achieved in efforts toward "a workable relationship between musical practice and the new experimental practices of early modern science" (p. 657).

June Barrow-Green tells the curious tale of Rolle's theorem from its first statement (1690) in the context *not* of calculus (which Rolle deeply distrusted) but of algebra, specifically the determination of "limits" of roots of polynomial equations. The surprisingly slow transition to the language of calculus was completed only by Hermite and by Serret, late in the 19th century. Niccolò Guicciardini chronicles the many attempts to pin down precisely the nature of Newton's achievement as a mathematician. Relevant are the initial reception of the *Principia*, the disputes over priority in the calculus, and 19th-century perspectives influenced by decades of successful application of Leibniz's version. Newton's own authorial and publication strategies are a recurrent theme.

Irina and Dmitri Gouzévitch describe the role of mathematics in Peter the Great's amazing program for the modernization of Russia. For Peter the key to success was mastery of theory, and essential to that purpose was the *method* of mathematics, the "quintessence of all sciences" (p. 355). Russia's leap in 30 years (1695–1725) across "the cognitive distance that separates elementary arithmetic from differential calculus" (p. 353) was aided by a "void" (p. 369) of pre-existing tradition and an absence of religious censorship. Massimo Mazzotti traces the tangled consequences of the establishment in Naples of a corps of civil engineers (1808). Attendant issues included the passage from semi-feudalism to modernity, the aspirations of the middle class, conservative opposition to an "ideology of progress" (p. 259), differing visions of mathematical education, and contrasting approaches to mathematics itself (analytic-algebraic versus synthetic-geometric). Mazzotti's analysis of the interplay of these factors is admirably subtle. Snezana Lawrence's article on the Balkans before World War I offers a "trilogy" (p. 177): the mathematical cultures of (i) the Ottoman Empire around 1900, (ii) that empire's Orthodox population (mostly Greek), and (iii) Serbia. This last section is enlivened by the "most famous" (p. 187) of Serbian mathematicians, Mihailo Petrović, whose "passion for [gypsy] music and fishing, and his approachable character, all conveyed an image of a bohemian and intellectual elite, at the core of which lay excellence in the study of mathematics" (p. 191).

David Aubin aims to "enrich our understanding" (p. 274) of 19th-century changes in mathematics by looking at its role in observatories. He discusses the position, physical and social, of mathematics and its users within those institutions, and then considers the observatory as a "locus of particular mathematical cultures" (p. 276), specifically geometry (Gauss), statistics (Quetelet), and celestial mechanics (Poincaré). Mary Croarken devotes her paper to the human computers who served astronomy, navigation and table-making in 18th- and 19th-century Britain. Concentrating on the National Almanac Office and the Greenwich Observatory, she sketches the nature, conditions and remuneration of these humble toilers' work, and she brings to life in some measure several individuals among them, of whom precious records survive.

Commendably, the editors have made room for no fewer than five papers on 20th-century developments. Tinne Hoff Kjeldsen pursues two themes, the increases in (i) abstraction and (ii) applications to "non-physical sciences" (p. 775). Both, she says (p. 756), led to "radical new interpretations" of pieces of mathematics previously considered unimportant—a claim which she illustrates by the emergence of the theory of convex sets and the creation of mathematical programming. Leo Corry provides what seems to a non-expert a capital history of the Bourbaki project, from its inception to its 1960s heyday and beyond, including an assessment of its achievement and its influence. Special attention is given to the central idea of a mathematical structure and, in the movement's later phase, the challenge posed to that concept by the rise of category theory.

The other three "recent" essays address social and cultural issues. Karen Hunger Parshall recounts the 19th- and 20th-century emergence of global connections among mathematicians sharing common values and goals. This internationalization occurred, she says (p. 86), "in the context both of the formation of professional communities in a historically conditioned, geopolitical world and of the development of a common sense of research agenda via the evolution of a nationally transcendent mathematical language". Reinhard Siegmund-Schultze begins his account of mathematics in the Third Reich with a survey of the subject's historiography, and moves in turn to the influence of Nazi ideology on mathematics before 1933, to mathematics under the Nazis—with Bieberbach as the central figure—and to the subsequent mass emigrations from Europe, "arguably the most important historical consequence of Nazi rule for mathematics" (p. 871). Jeremy Gray takes on the meatiest topic in all of these pages, examining the extent to which the changes in mathematics around 1900 can be said to parallel contemporary trends in painting (exemplified here by Picasso), music (Schoenberg) and literature (Joyce). From surveys of algebra, analysis and geometry Gray concludes that we can indeed usefully transfer the term "modernist" from the arts to mathematics, common features including "a strong emphasis on novelty of form and on new criteria for appreciation, which were much more internal" (p. 680). It is pleasant to report the proof offered by this essay that weightiness of theme need not preclude deftness of style.

Obviously such potted précis can barely even *hint* at the wealth of fact, argument and insight on offer in these crowded pages. Moreover space constraints and the—to put it very kindly—uneven character of my own competence in this daunting diversity of subjects rule out any evaluation here of individual papers' substance. Let experts judge! But some thoughts on the book as a whole may be in order. Robson and Stedall have maintained high scholarly standards; for example absolutely everything is translated into English from other languages, but with the originals in same-page footnotes. The writing is generally very good. The editors hoped (p. 1) that their contributors would be "engaging and accessible", and the second of these desiderata has been delivered across the board, the first with only slightly less consistency. Amusingly or poignantly, according to taste, the most magnificent sentence in all of these 21st-century pages is the work of William Whewell, writing in 1837 (it is quoted on p. 717). But the prose here is always at least competent, and often much better than that. There are, to be sure, *many* mistakes in English—in particular it seems that at production time OUP's stockroom must have been severely short of hyphens—but virtually all of these slips are minor and do not impede understanding.

Thus a summarizing judgment on this massive volume can enthuse in every direction but one. Is it a handbook, in any usual sense of the word? No way. But is it a splendid, something-for-everybody treasure-trove of interesting, informative, challenging, well written testaments to the variety and vigor of history of mathematics in our time? No question.

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Aspects of the Astrolabe: 'Architectonica Ratio' in Tenth- and Eleventh-century Europe
By Arianna Borrelli. Stuttgart (Franz Steiner Verlag). 2008. ISBN 978-3-515-09129-9.272 pp. €44.00.

Studying ancient and medieval science can be a daunting challenge. Points of view, perspectives, categories, even meanings of words can subtly and indiscernibly be incommensurable with our own. Reading a medieval text can be an exercise in first contact with an alien culture; one is never sure if the meaning one sees is the meaning that is really there. Now add to that the difficulty of studying not a text, but a scientific instrument. What remains to us might be quite different from what was there at the time. Instruments disappear; texts do not—or perhaps it is the other way around. Who is to know? How then does one reconstruct with any reliability what the earliest practitioners actually thought and knew about the device? So much of what went on is likely to have happened in conversations, observing sessions, actually building and using it: all ephemeral tasks that could never make it down to us.

Arianna Borrelli's *Aspects of the Astrolabe* is a careful attempt to wade into these dark waters: in particular, into the arrival and early experiences of the astrolabe in Latin Europe in the 10th and 11th centuries. The earliest texts appear muddled, confused, poorly written, and even often wrong. But are we reading them correctly? The task facing us here is akin to trying to reconstruct the history of 20th-century technology using only a single poorly-translated instruction manual for a DVD player. Borrelli attempts to treat the fragmentary evidence of the early Latin astrolabe with dignity: to consider, as well as possible, the unwritten aspects of astrolabe culture that might help us to make sense of the seemingly primitive surviving manuscripts.

Borrelli's two main theses are: (a) that "the assimilation of astrolabe knowledge in Latin Europe was the result of a combination of written and non-written, verbal and non-verbal strategies of knowledge transfer" (p. 21); and (b) "that high medieval astrolabe studies could be linked to an image of knowledge in which the material effects of what we today regard as 'applied mathematics' were epistemologically relevant" (p. 22). The words here are chosen carefully: an "image of knowledge", for instance, represents a culture's stance on what knowledge is, what kinds of knowledge are legitimate, the means by which this knowledge translates into statements about nature, and so on.